Forefront-Background Separation From Video Clips via Motion-Assisted Matrix Restoration

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Abstract—Separation of video clips into foreground and background components is a useful and important technique, making recognition, classification and scene analysis more efficient. In this paper, we propose a motion-assisted matrix restoration (MAMR) model for foreground-background separation in video clips. In the proposed MAMR model, the backgrounds across frames are modeled by a low-rank matrix, while the foreground objects are modeled by a sparse matrix. To facilitate efficient foreground-background separation, a dense motion field is estimated for each frame, and mapped into a weighting matrix which indicates the likelihood that each pixel belongs to the background. Anchor frames are selected in the dense motion estimation to overcome the difficulty of detecting slowly-moving objects and camouflages. In addition, we extend our model to a robust MAMR model (RMAMR) against noise for practical applications. Evaluations on challenging datasets demonstrate that our method outperforms many other state-of-the-art methods, and is versatile for a wide range of surveillance videos.

Index Terms—Background segmentation/subtraction, motion detection, optical flow, matrix restoration, video surveillance.

I. INTRODUCTION

VIDEOS have become the basic representation of interesting scenes and events, and are widely used in many areas such as entertainment, public-security surveillance, healthcare. As a consequence, video analysis is of crucial importance to mine interesting information from mass data [1]–[3]. Separation of foreground and background [4]–[7] is to divide a video clip into two complementary components: the background and the foreground, which has become a useful technique for video analysis in many applications such as motion detection [8], [9], object recognition [10], and video coding [11].

For accurate foreground-background separation, there are many tough problems arising from the practical applications, for example, illumination changes: the background has intensity variations due to lighting changes [12]; camouflage: slowly-moving objects are difficult to identify, resulting in wrong classification; noise: video signals are usually contaminated by various types of noise. Previous methods, such as Gaussian mixture model (GMM) [13], non-parametric kernel density estimation [14], and methods based on robust principle component analysis (RPCA) [15], have addressed some of these factors and made significant progress (detailed in Sec. II), but more research work is still necessary to achieve more accurate separation of foreground and background components in video clips.

In this paper, we propose a new foreground-background separation method via motion-assisted matrix restoration (MAMR). Figure 1 illustrates the work flow of our method. The main idea is to incorporate motion information into the matrix recovery framework to facilitate the separation of the foreground and the background. To this end, a dense motion field is first estimated for each frame against an anchor frame, and mapped into a weighting matrix which indicates the likelihood that each pixel belongs to the background. Anchor frames are selected in the dense motion estimation process to overcome the difficulty in detecting slowly-moving objects and camouflages. The separation problem is then formulated into a motion-assisted matrix restoration (MAMR) model with the weighting matrix. The model is solved by the alternating direction method under the augmented Lagrangian multiplier (ADM-ALM) framework. Then we estimate the foreground using our background subtraction technique. In addition, we extend our model to a robust MAMR model (RMAMR) for practical applications. Experiments show that our method achieves consistently better performance than many state-of-the-art methods on various datasets with different characteristics (e.g., motions, lighting conditions, and noise).

The rest of this paper is organized as follows. In Sec. II, we present a brief overview of related work. Sec. III presents the formulation of weighting matrix, the MAMR model, and the extended RMAMR model; we further develop the ADM-ALM algorithm to solve the proposed models in this section. Experimental results and analysis are given in Sec. IV, and conclusions are drawn in Sec. V.

II. RELATED WORK

Background extraction and foreground detection techniques can be divided into two categories: local methods and global methods. We give a brief overview for these two categories.

A. Local Methods

Local methods usually operate on each pixel individually. Some simple methods, including running Gaussian average
temporal median filtering [17], and first-order low-pass filtering [18], in some cases offer satisfactory accuracy with high processing speed, but have difficulties to deal with backgrounds with multi-modal intensity distributions. To model the multi-modality background, methods based on Gaussian mixture model (GMM) [13], [19], [20] achieve significant improvements, but are still difficult to handle challenging video clips with varying lighting conditions and/or dynamic backgrounds. The non-parametric model based on kernel density estimation [14] is more robust to rapid variations of backgrounds. ViBe [21], which is also a non-parametric model method, introduces a random strategy to update the background values. Hofmann et al. [22] proposed the pixel-based adaptive segmenter (PBAS) by assigning adaptive randomness parameters. Besides, Godbehere et al. [23] introduced a pixel-wise Bayesian segmentation algorithm that identifies foreground objects from an inferred foreground model and an estimated background. Yao et al. [24] introduced a robust multi-layer background subtraction technique which takes advantages of local texture features represented by local binary patterns and photometric invariant color measurements in RGB color space. Self-organization background subtraction (SOBS) [25] proposed by Maddalena et al. learned background motion with a self-organizing neural network, and obtains impressive detection results for scenes with gradual illumination variations. The $\Sigma - \Delta$ motion detection filter [26] is applicable to embedded systems, but compromises on the detection accuracy to some extent.

Generally, local methods enjoy the simplicity in design and implementation, but the resulting segmentation map often suffers from spatial inconsistency. Also, these techniques are sensitive to perturbations (e.g., noise, illumination variations), and yield misclassifications around boundaries between the background and foreground.

**B. Global methods**

In contrast to local methods, global methods exploit more spatial correlation information. Markov random field (MRF) based methods are frequently used in background extraction for integrating spatial or spatial-temporal information. Yue et al. [27] presented a time dependent MRF model with multi-resolution spatiotemporal pyramids. More recently, based on fuzzy GMM and MRF, Zhao et al. [28] introduced the spatiotemporal constraints into the model to deal with dynamic backgrounds.

Principal component analysis (PCA), widely used in classic data analysis, is also powerful in background modeling. Seki et al. [29] trained a PCA for each block-volume over time, and determined the belonging (to the background or foreground) of each block by measuring its projection to the trained PCA. The eigenspace model [30] is proposed to detect moving objects. Using blocks as basic units, PCA-based methods are prone to misclassifying pixels at foreground-background boundaries.

Robust PCA (RPCA) [15], a well-known extension of PCA, is able to efficiently exploit the underlying low-rank structure in the data even in the presence of large errors or outliers. Recently, many background and foreground separation methods based on RPCA have been developed [31]–[36]. Gao et al. [31] introduced a two-pass RPCA combining with motion saliency estimation to detect foreground. Guyon et al. [32] proposed an adapted $\ell_{2,1}$ norm to model the sparse component, which satisfies the ad hoc block-sparse hypothesis. Zhou et al. [9] improved previous RPCA-based methods by using $\ell_0$ norm instead of $\ell_1$ norm to model the sparse component, and incorporating contiguity prior using MRF to make the foreground objects spatially consistent. Bouwmans et al. [33] presented a comprehensive review on RPCA-PCP based methods [34]–[36] for testing and ranking existing algorithms for foreground detection.

In general, many methods have been developed using the framework of sparse representation and rank minimization. However, previous methods are motion-unaware and would introduce smearing artifacts when handling slow motion and motionless foreground (camouflages). To be aware of motions, our work encodes motion information into the low-rank and sparse recovery model by a weighting matrix, which is distinct from the recent work in [9] that improves RPCA by imposing smoothness of the foreground component. The proposed
A. Motivation

In these rows are thus dense, which does not meet the sparse assumption. Therefore, previous RPCA-based methods present smearing artifacts around regions with slow motions or even camouflage. To overcome this shortcoming, it is desirable to find a smart way to let the model be aware of slow motions of foreground objects, which motivates us to propose a motion-assisted matrix restoration (MAMR) model for background-foreground separation.

B. Framework

The key idea of our MAMR method is to assign to each pixel a likelihood that it belongs to the background based on the estimated motion at that pixel. The background is to be extracted from $K$ frames of a surveillance video clip denoted by $\{i_k\}_{k=0}^{K-1}$ of size $M \times N$. For easy mathematical manipulation, let $i_k$ be the vector form of frame $i_k$ with the size $MN \times 1$. Then, we represent the frame sequences with matrix $D = [i_0, i_1, ..., i_{K-1}]$ of size $MN \times K$. The recovered background component and foreground component in $D$ are denoted by $B$ and $F$ respectively. The aim is to separate $B$ and $F$ from $D$. Denote a matrix, named weighting matrix, by $W$ whose elements represent the confidence levels that corresponding pixels in $D$ belong to the background.

We propose to solve the foreground-background separation problem by solving the following optimization formula:

$$\min_{B,F} \|B\|_s + \lambda \|F\|_1, \quad \text{subject to} \quad W \odot D = W \odot (B + F), \quad (1)$$

where $\|\cdot\|_s$ and $\|\cdot\|_1$ denote the nuclear norm (sum of singular values) and $\ell_1$ norm of a matrix, respectively, and “$\odot$” denotes element-wise multiplication of two matrices. Like previous methods, it is reasonable to assume the background as motionless in most practical surveillance applications (otherwise a global motion should be compensated). Under this assumption, any area with motion should not be considered as a part of background. Therefore, the weighting matrix $W$ is constructed from motion information (see Sec. III-C). Model (1) extends the classic matrix recovery model by taking the reliability of observed data into consideration. By incorporating motion information, areas dominated by slowly-moving objects are suppressed while the background that appears in only a few frames has more chances to be recovered in the final results.

III. BACKGROUND MODELING VIA MOTION-ASSISTED MATRIX RESTORATION

A. Motivation

The RPCA-based methods decompose the observed matrix (constructed by shaping each frame into a vector, and put vectors corresponding to successive frames as columns in the matrix) into two components. The low-rank component corresponds to the stationary background, while the sparse component represents the moving objects. Generally, the RPCA model fits well the background and foreground characteristics when foreground objects move fast: the latent background should be the same for all the frames within a scene (hence low-rank) and the foreground scatters in the spatio-temporal volume of the video clip (hence sparse). However, this prior assumption can be violated when the foreground occupies a large portion of the scene densely. Figure 2 shows a video clip containing two cars. The right car stays motionless all the time, and hence belongs to the background. As the left car moves slowly (belonging to foreground), background pixels are occluded by the car in many frames. In the observed matrix, each row corresponds to one pixel to be recovered in the background image, and the elements in a row are pixels from the background or the foreground along the temporal direction. As shown in Fig. 2 (c), many rows are dominated by the intensities of the left car, and the foreground components in these rows are thus dense, which does not meet the sparse assumption. As a result, the foreground information would leak into the recovered background component.

method also preserves the spatial smoothness of the foreground component to some extent as the used optical flow estimator [37] has considered the smoothness of the motion field (hence the foreground). Our successful attempt might serve as a good starting point to exploit the incorporation of more complex motion models or other clues into the low-rank and sparse recovery framework for foreground detection.

B. Framework

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$$\min_{B,F} \|B\|_s + \lambda \|F\|_1, \quad \text{subject to} \quad W \odot D = W \odot (B + F), \quad (1)$$

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C. Weighting Matrix Construction From Motion Information

Usually, the optical flow is computed pair-wise between two consecutive frames. However, for practical video clips, moving objects may move slowly or even stay motionless across many frames, i.e. camouflage, which are difficult to detect by optical flow. As shown in Fig. 6(a) (top row in red rectangle), the bag put on the carton by the left man is a camouflage across many frames. The optical flow between two adjacent frames is not sufficient to determine whether it belongs to foreground or background, resulting in misclassification. To remedy this problem, for each frame, we find a proper reference frame (called anchor frame, not necessarily the adjacent one) that differs from the current frame even in regions containing slowly moving foreground objects or even camouflages. Then, we estimate motion information for each frame referring to its
nearest anchor frame. Finally, we map the motion field into a weighting matrix.

1) Dense Motion Estimation With Anchor Frame Selection:
For a single video, we set the first frame \(i_0\) as the initial anchor frame. The remaining anchor frames are automatically selected according to the difference against the previous nearest anchor frame. To this end, the difference between the current frame \(i_k\) and the previous nearest anchor frame \(i_{anchor}\) is calculated for each frame. The difference \(e_k\) is defined as mean absolute difference (MAD) between two frames:

\[
e_k = \frac{\sum_{m,n\in M\times N} |i_{k,m,n} - i_{anchor,m,n}|}{M \times N}
\]

where \(m, n\) are the two-dimension pixel indexes in a frame. If the difference is larger than a threshold \(T\), this frame is selected as a new anchor frame.

For each frame, we use the optical flow method in [37] to extract a dense motion field \((\alpha^x_k, \alpha^y_k)\) between current video frame \(i_k\) and its previous nearest anchor frame, where \(\alpha^x_k\) and \(\alpha^y_k\) are the horizontal component and vertical component of the motion field, respectively. Both \(\alpha^x_k\) and \(\alpha^y_k\) are in the vector form in the same organization as \(i_k\). Note that \(T\) should be chosen appropriately: too large a threshold would lead to few anchor frames, while too small a threshold would result in underestimation of motion (hence smearing artifacts around frame. To this end, the difference between the current frame \(i_k\) and the previous nearest anchor frame \(i_{anchor}\) is calculated for each frame. The difference \(e_k\) is defined as mean absolute difference (MAD) between two frames:

\[
e_k = \frac{\sum_{m,n\in M\times N} |i_{k,m,n} - i_{anchor,m,n}|}{M \times N}
\]

For each frame, we use the optical flow method in [37] to extract a dense motion field \((\alpha^x_k, \alpha^y_k)\) between current video frame \(i_k\) and its previous nearest anchor frame, where \(\alpha^x_k\) and \(\alpha^y_k\) are the horizontal component and vertical component of the motion field, respectively. Both \(\alpha^x_k\) and \(\alpha^y_k\) are in the vector form in the same organization as \(i_k\). Note that \(T\) should be chosen appropriately: too large a threshold would lead to few anchor frames, while too small a threshold would result in underestimation of motion (hence smearing artifacts around slowly-moving objects and misclassification of camouflages).

2) Motion-to-Weight Mapping: In the proposed model, the weighting matrix \(W\) is constructed from the extracted dense motion field. We use the sigmoid function to map the motion field \((\alpha^x_k, \alpha^y_k)\) into the weighting matrix. We define \(o^x\) of size \(MN \times K\) as the matrix form of horizontal motion fields for all frames in \(D\) by stacking \(o^x_k\), \(k = 0, 1, \ldots, K - 1\) as columns. Similarly, \(o^y\) is defined for vertical motion fields. The weighting matrix \(W\) is constructed as follows:

\[
w_{jk} = \frac{1}{1 + \exp\left(\alpha - \sqrt{\left(o^x_{jk}ight)^2 + \left(o^y_{jk}\right)^2 + \beta}\right)}
\]

where \(\alpha\) and \(\beta\) are the parameters of the sigmoid function which control the fitting slope and phase, respectively. \(\beta\) is chosen according to the average intensity of the motion field. Note that \(\alpha\) is a crucial parameter to shape the importance of motion information, as shown in Fig. 3: if \(\alpha\) is zero, the weighting matrix \(W\) is equal to 0.5 in all elements, and Model (1) turns into traditional RPCA-based method; As \(\alpha\) increases, the slope of sigmoid function becomes steeper; when \(\alpha\) takes very large values, e.g. 10, the sigmoid function will become approximately a step function, while \(W\) also turns into a binary matrix, i.e., \(W \in \{0, 1\}^{MN \times K}\).

Specifically, the weighting matrix \(W\) is degraded from Formula (3) to the following binary mask:

\[
w_{jk} = \begin{cases} 0, & \sqrt{(o^x_{jk})^2 + (o^y_{jk})^2} \geq \beta, \\ 1, & \text{otherwise}. \end{cases}
\]

With such weighting, Model (1) becomes the following matrix completion model:

\[
\min_{B, F} ||B||_* + \lambda||F||_1, \text{ subject to } P_{\Omega}(D) = P_{\Omega}(B + F),
\]

where \(\Omega\) denotes the linear subspace of entries in the observed matrix that belong to background for sure, and \(P_{\Omega}(\cdot)\) is the associated projection operator.

D. The ADM-ALM Algorithm to Solve the MAMR Model

The MAMR model is essentially a convex optimization problem that can be solved by ADM-ALM method [38], [39]. The idea of ALM framework is to convert the original constrained optimization problem (1) to the minimization of the augmented Lagrangian function:

\[
L(B, F, Y, \mu) = ||B||_* + \lambda||F||_1 \\
+ \langle Y, W \odot (D - B - F) \rangle \\
+ \frac{\mu}{2} ||W \odot (D - B - F)||^2_F,
\]

where \(\mu\) is a positive constant, \(Y\) is the Lagrangian multiplier. \(\langle \cdot, \cdot \rangle\) denotes the matrix inner product, and \(||\cdot||_F\) denotes the matrix Frobenius norm.

Instead of optimizing \(B, F\) and \(Y\) simultaneously, the ADM solves \(B, F\) and \(Y\) alternatingly:

\[
\begin{align*}
F_{j+1} &= \arg\min_F \frac{\mu_j}{2} ||W \odot (D - B_j - F)||^2_F, \\
B_{j+1} &= \arg\min_B ||B||_* - \langle Y_j, W \odot B \rangle \\
Y_{j+1} &= Y_j + \mu_j W \odot (D - B_{j+1} - F_{j+1}), \\
\mu_{j+1} &= \rho\mu_j,
\end{align*}
\]

The solution of \(F_{j+1}\) has the following closed-form

\[
F_{j+1} = \text{shrink} \left( \frac{1 - Y_j + W \odot (D - B_j)}{\mu_j}, \frac{\lambda}{\mu_j} \right)
\]

where \(\text{shrink}(\cdot, \cdot)\) is the soft-thresholding function defined as:

\[
\text{shrink} (X, t) = \text{sign} (X) \max (\text{abs} (X) - t, 0)
\]

The soft-thresholding operator applies on the matrix \(X\) in an element-wise manner.
The solution of $B_{j+1}$ in (7) does not have a closed-form solution, and we resort to the accelerated proximal gradient algorithm [40] given as:

\[
\begin{align*}
(U_t, S_t, V_t) &= 
\begin{cases}
(U_t, S_t, V_t) = 
\begin{aligned}
&\text{svid}\left( \frac{1}{\mu_j} Y_j + W \circ (D - Z_t) - F_{j+1} + Z_t \right), \\
&B_{j+1} = U_t \text{shrink} \left( S_t, \frac{1}{\mu_j} \right) V_t^T, \\
&Z_{t+1} = B_{j+1} + \frac{l_t - 1}{l_t} (B_{j+1} - B_j), \\
l_{t+1} = 0.5(1 + \sqrt{1 + 4l_t^2}),
\end{aligned}
\end{cases}
\end{align*}
\]

(10)

where $l_t$ is a positive sequence with $l_1 = 1$, svid$(\cdot)$ denotes the singular value decomposition of a matrix.

The entire algorithm to solve problem (1) is summarized as Algorithm 1. In the ADM-ALM framework, the sub-problems are not necessarily solved exactly as long as the approximated solutions reduce the cost of Lagrangian function, which is therefore called inexact ALM [41]. Allowing inexact approximation of the sub-problems actually reduces overall computational complexity as the inner-loop iterations require considerable amount of computation to reach convergence. In our implementation, the inner loop for solving $B_{j+1}$ has only one iteration for acceleration.

The solution of Model (1), denoted by $(B^*, F^*)$, is obtained after the convergence of the iterative procedure: $B^*$ contains a background component for each frame, while $F^*$ provides a foreground component for each frame. We take the average of all columns in $B^*$ as the final recovered background image $b$. Note that the $\ell_1$ regularizer essentially describes signals that conform the Laplacian distribution. As a result, $F^*$ contains not only the desired foreground components but also noise leaked from background areas (due to the low-rank regularization). Therefore, we do not use $F^*$ as the foreground solution.

Rather we extract foreground using the background subtraction approach with the recovered background $\bar{b}$ (detailed in Sec. III-E).

**Algorithm 1** ADM-ALM algorithm for the MAMR model.

**Input:** $D \in \mathbb{R}^{MN \times K}$, $W \in \mathbb{R}^{MN \times K}$, $\lambda > 0$, $\rho > 0$, $\mu > 0$;

**Initialize:** $F = 0$, $B_1 = 0$, $Y_1 = 0$;

while not converged do

$F_{j+1} = \text{shrink} \left( \frac{1}{\mu_j} Y_j + W \circ (D - B_j), \frac{1}{\mu_j} \right)$;

$t_1 = 1$, $Z_1 = B_j$, $B_{j+1} = B_j$;

while not converged do

$(U_t, S_t, V_t) = 
\begin{cases}
(U_t, S_t, V_t) = 
\begin{aligned}
&\text{svid}\left( \frac{1}{\mu_j} Y_j + W \circ (D - Z_t) - F_{j+1} + Z_t \right), \\
&B_{j,l+1} = U_t \text{shrink} \left( S_t, \frac{1}{\mu_j} \right) V_t^T, \\
&Z_{l+1} = B_{j,l+1} + \frac{l_t - 1}{l_t} (B_{j+1} - B_{j,l}); \\
l_{t+1} = 0.5(1 + \sqrt{1 + 4l_t^2}), l = l + 1;
\end{aligned}
\end{cases}
\end{align*}

end while

$B_{j+1} = B_{j,l+1}$;

$Y_{j+1} = Y_j + \mu_j W \circ (D - B_{j+1} - F_{j+1})$;

$\mu_{j+1} = \rho \mu_j, j = j + 1$;

end while

**Output:** $(B_j, F_j)$;

**E. Foreground Separation with Background Subtraction**

Denote by $\bar{f}_k$ the foreground image for frame $i_k$. The intensity value of $\bar{f}_k$ at pixel $x$, denoted by $\bar{f}_k(x)$, is determined as:

\[
\bar{f}_k(x) = \begin{cases} 
G(x) \cdot \frac{\sum_{x \in N_b} |G(x) - |G(x)| \cdot |G(x)|^2|}{|N_b|} > t + \sigma, \\
0 \quad \text{otherwise}
\end{cases}
\]

(11)

where $N_b$ is the neighborhood of size $\omega \times \omega$ around $x$. $|N_b|$ is the number of pixels in $N_b$; $\sigma$ represents the level of noise variations in $i_k$; $t$ is defined as:

\[
t = \frac{\sum_{x \in \Phi} |i_k(x) - b(x)|}{|\Phi|},
\]

(12)

where $\Phi$ is the set of pixels which contains non-zero values in $|i_k(x) - b(x)|$; $|\Phi|$ is the number of non-zero pixels in the set $\Phi$. By thresholding the average background subtraction image value over a small window, the outliers can be removed while the true foreground pixels are retained. For comparison in the experimental section, we convert the foreground image $\bar{f}_k$ into a binary map by replacing the non-zero values in $\bar{f}_k$ with 255.

**F. Robust MAMR**

In real applications, noise is quite ubiquitous. Usually, the data matrix is seriously damaged in some elements, while all of the elements would receive some lightweight noise pollution. Though the $\ell_1$ norm can separate the intensive sparse errors from the intrinsic low-rank data matrix, it cannot deal with dense noise distributed over the whole frames. So, we propose a robust MAMR (RMAMR) model. We use the Frobenius norm to model dense noise. Denote by $G$ the error matrix of dense noise, the model can be formulated as follows:

\[
\min_{B, F, G} \|B\|_* + \lambda \|F\|_1 + \gamma \|G\|_F^2,
\]

subject to $W \circ D = W \circ (B + F + G)$,

(13)

where $\gamma$ is a positive constant, and $\| \cdot \|_F$ denotes the matrix Frobenius norm. The augmented Lagrangian function of problem (13) is given by

\[
L(B, F, G, Y, \mu) = \|B\|_* + \lambda \|F\|_1 + \gamma \|G\|_F^2 + \langle Y, W \circ (D - B - F - G) \rangle + \frac{\mu}{2} \|W \circ (D - B - F - G)\|_F^2.
\]

(14)

Note that the difference between Model (1) and Model (13) is the introduction of the quadric term of $G$. The solutions of $B$ and $F$ subproblems are similar to those in Model (1). So, we only present the solution of $G$-subproblem:

\[
G_{j+1} = \arg \min_G \|G\|_F^2 - (Y_j + \mu_j W \circ (D - B_j - F_{j+1})) \cdot \mu_j,
\]

(15)

The solution of $G$ has the following closed-form:

\[
G_{j+1} = \frac{1}{\mu_j + 2\gamma} (Y_j + \mu_j W \circ (D - B_j - F_{j+1})).
\]

(16)
The entire algorithm to solve problem (13) is summarized as Algorithm 2.

**Algorithm 2** ADM-ALM algorithm for the RMAMR model.

**Input:** $D \in \mathbb{R}^{MN \times K}$, $W \in \mathbb{R}^{MN \times K}$, $\lambda > 0$, $\rho > 0$, $\gamma > 0$, $\mu > 0$;

**Initialize:** $F_1 = 0$, $G_1 = 0$, $B_1 = 0$, $Y_1 = 0$;

while not converged do

$F_{j+1} = \text{shrink}(\frac{1}{\rho_j} (Y_j + W \odot (D - B_j - G_j)), \frac{1}{\mu_j})$;

$G_{j+1} = \frac{1}{\rho_j} (Y_j + \mu_j W \odot (D - B_j - F_{j+1}))$;

$t_j = 1$, $Z_j = B_j$, $B_{j+1} = B_j$;

while not converged do

$(U_t, S_t, V_t) = \text{svd}(\frac{1}{\mu_j} Y_j + W \odot (D - Z_t) - F_{j+1} - G_{j+1} + Z_t)$;

$B_{j,t+1} = U_t \text{shrink}(S_t, \frac{1}{\mu_j}) V_t^T$;

$Z_{l+1} = B_{j,l+1} + \frac{t_{l+1}}{t_l} (B_{j,l+1} - B_{j,l})$;

$t_{l+1} = 0.5(1 + \sqrt{1 + 4(t_l^2)})$,

end while

$B_{j+1} = B_{j,l+1}$;

$Y_{j+1} = Y_j + \mu_j W \odot (D - B_{j+1} - F_{j+1} - G_{j+1})$;

$\mu_{j+1} = \rho \mu_j$, $j = j + 1$;

end while

**Output:** $(B_j, F_j, G_j)$;

IV. EXPERIMENTAL RESULTS

In this section, we first present the setting of parameters in our algorithm (Sec. IV-A), and introduce test video clips and performance metrics used in our paper (Sec. IV-B). Then we investigate the parameters in weighting matrix construction that affect the recovery performance (Sec. IV-C), and compare different combining options to evaluate the impact of each module in our model (Sec. IV-D). Next, we compare our MAMR model with other state-of-the-art methods on challenging datasets in terms of background extraction (Sec. IV-E) and foreground detection (Sec. IV-F). In addition, we show the robustness to noise of our RMAMR model in Sec. IV-G. The running time is reported in Sec. IV-H.

In this paper, our method is compared with thirteen (13) methods: visual background extractor (ViBe) [21], self-organizing background subtraction (SOBS) [25], Gaussian mixture model (GMM) [13], statistical Bayesian segmentation and tracking (SBST) [23], pixel-based adaptive segmenter (PBAS) [22], fuzzy background modeling method (FBM) [28], Gaussian mixture model of Laurence Bender (LGB) [42], multi-layer background subtraction (MBS) [24], principal component pursuit (PCP) [15], outlier Pursuit (OP) [34], semi-soft GoDec algorithm (SSGoDec) [35], sparse Bayesian for low-rank matrix estimation (SBL) [36], and DEtecting Contiguous Outliers in the Low-rank Representation (DECOLOR) [9]. The codes for PCP, OP, SSGoDec, SBL are available at the project website [33], [43]. The codes for ViBe, GMM, SOBS, and DECOLOR are provided by the authors. The remaining methods are publicly available from Bgslibrory [44]. Since GMM, SOBS, LBG, MBS, and the RPCA-based methods can generate both the background image and the binary foreground map, we compare the extracted backgrounds with these methods in (Sec. IV-E and Sec. IV-G). For all above algorithms, we seek optimal parameters around initial parameters published by the authors for fair comparison.

All the results are available in the project website\(^1\). We direct interested readers to the website for more visual comparison results.

A. Parameter Setting

The parameters in our method fall into two categories: parameters ($\rho, \mu$) that affect algorithm convergence and parameters ($T$, $\alpha$, $\beta$, $\lambda$, $\gamma$, $\sigma$, and $\omega$) that influence the performance.

1) Convergence parameters: $\mu$ is increasing during iterations from a small initial value $1/\text{LSV}(F)$, where $\text{LSV}(\cdot)$ takes the largest singular value of the operand matrix [38], [39]. In terms of $\rho$, too large a value would lead to unsatisfactory result, while too small one would slow the convergence rate of the algorithm. So we empirically set $\rho = 2$ for all the datasets.

2) Performance parameters: $\sigma$ and $\omega$ are related to foreground detection. Thresholding factor $\sigma$ in (11) depends on the level of noise and the average color difference between foreground pixels and background pixels in a video clip. It is chosen between the range $[15, 35]$ for all the video clips (see Sec. IV-B). The neighborhood size $\omega$ in (11) is fixed at $3 \times 3$.

The parameters $T$, $\alpha$, $\beta$ control the construction of weighting matrix. For each frame, $T$ is adaptively set according to the average motion intensity over the previous processed $K$ frames: $T = 1.3 \sum_{k=K}^{K} e_k / K$. If $e_k$ is larger than $T$, the current frame is selected as a new anchor frame. $\beta$ controls the turning point of the sigmoid function, and reflects the motion level beyond which is considered significant. In our implementation, $\beta$ is chosen as the average intensity of the motion field, which is satisfactory for various datasets. Usually, $\alpha$ is set at a large value for a binary weighting matrix. The detailed discussion of $\alpha$ is given in Sec. IV-C.

The parameters $\lambda$ and $\gamma$ adjust the importance between low-rank term, sparse term, and noise term. In the noise-free case, our MAMR model set $\lambda = 10$, a large value that emphasize the importance of sparse regularization. In noisy case, our RMAMR model set $\lambda = 1$ and $\gamma = 1$ for the tested noise level.

B. Test Datasets and Performance Metrics

For comprehensive evaluation, we test our method on ten (10) video clips from ChangeDetection dataset (CDnet) [45] [46], and other two typical video clips Monitor and Train. CDnet contains six video categories with four to six video clips in each category. We choose the whole video clips from the category Dynamic Background, including Boats, Canoe, Fall, Fountain01, Fountain02, and Overpass; and pick one representative from each of other four categories, i.e., Office from Baseline, Winterdrive (Winter) from Intermittent Object Motion, Boulevard from Camera Jitter, and PeopleInShade (Shade) from Shadow. We pick continuous 200 frames from each.

\(^1\)http://cs.tju.edu.cn/faculty/likun/projects/bf_separation/index.htm
dataset in the experiment. The key information of these twelve datasets is summarized in Table I. Each of these datasets may include various kinds of motions, lighting variations, camera jitter, camouflages, shadows, dynamic backgrounds, or the combination of them.

For objective evaluation in background extraction, ground-truth background images for static videos are created by averaging the background frames (without foreground included), which are manually picked from the sequence (shown in Fig. 4). We use the peak signal-to-noise ratio (PSNR) to measure the quality of extracted backgrounds against their ground-truth. Datasets with dynamic backgrounds are difficult to acquire the quality of extracted backgrounds against their ground-truth. Therefore, we use the peak signal-to-noise ratio (PSNR) to measure the objective performance of different algorithms by three metrics, namely Recall (Re), Precision (Pre), and F-measure \((F_1)\):

\[
\begin{align*}
\text{Recall} &= \frac{tp}{(tp + fn)} \\
\text{Precision} &= \frac{tp}{(tp + fp)} \\
F_1 &= \frac{2 \times \text{Recall} \times \text{Precision}}{(\text{Recall} + \text{Precision})}
\end{align*}
\]

where \(tp\) (true positive) represents correctly classified foreground pixels, \(fn\) (false negative) denotes the number of foreground pixels incorrectly classified as background, \(fp\) (false positive) stands for the total number of background pixels incorrectly classified as foreground. \(F_1\) gives the percentage of correctly detected foreground pixels among all detected foreground pixels. \(Recall\) weights the percentage of correctly detected foreground pixels among the total number of foreground pixels. \(F\)-measure is the weighted harmonic mean of \(Precision\) and \(Recall\), which measures the overall detection quality of an algorithm. For all the three metrics, the higher the value is, the better the performance it has.

C. Effect of \(\alpha\) in Motion-to-Weight Mapping

Note that \(\alpha\) is a crucial parameter to map the motion field \((o_x^k, o_y^k)\) into weighting matrix \(W\). We sample five values of \(\alpha\), i.e., 0, 0.5, 1.5, 3, and 10 (which generates a nearly binary matrix) to investigate how \(\alpha\) affect the recovery performance. A linear mapping is also tested between \(W\) and \((o_x^k, o_y^k)\). Fig. 5 shows our objective results on recovered backgrounds. As \(\alpha\) increases, the recovered performance gets better for each video clip, and reaches the highest PSNR when \(\alpha\) equals to 10 (approximately binary weight). This trend is particularly significant for Monitor and Train, because Monitor contains a slowly-walking men while the runaway thief occupies most space of the picture across many frames in Train.

Table I

<table>
<thead>
<tr>
<th>Name</th>
<th>Characteristic</th>
<th>Sec.IVC</th>
<th>Sec.IV D</th>
<th>Sec.IV E</th>
<th>Sec.IV F</th>
<th>Sec.IV G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office</td>
<td>Slowly-moving (man)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Winter</td>
<td>Camouflage (right car), lighting variation</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Boulevard</td>
<td>Camera jitter, fast-moving (cars)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Shade</td>
<td>Periodic motion, shadow (man)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Monitor</td>
<td>Periodic motion, slowly-moving</td>
<td></td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Train</td>
<td>Wagging (train), lighting variation</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Boats</td>
<td>Dynamic background (shimmering water)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Canoe</td>
<td>Dynamic background (shimmering water)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Fall</td>
<td>Dynamic background (waving tree)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Fountain01</td>
<td>Dynamic background (spring)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Fountain02</td>
<td>Dynamic background (spring)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Overpass</td>
<td>Dynamic background (waving tree)</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Fig. 4. Ground-truth background images for Office, Winter, Shade, Monitor, Train, Fall (enclosed by blue lines), and dynamic backgrounds excluding foregrounds for Boulevard, Boats, Canoe, Fountain01, Fountain02, Overpass (enclosed by yellow lines).

Fig. 5. Objective comparison with different values of \(\alpha\) for recovered backgrounds on five video clips (static backgrounds). The values are computed against ground-truths in PSNR.
Recall $F_1$ Pre OF4 + RPCA
(a) (b) (c) (d) (e)
0.7 0.74 0.8 0.7 0.73
0.6 0.71 0.76 0.78 0.78 0.81 0.81 0.83 0.85

Comparing the results of OF+RPCA and OF+AFS+RPCA, we observe that the performance would decline if the AFS is excluded, which demonstrates the effectiveness of AFS to detect moving objects.

TABLE II
QUANTITATIVE FOREGROUND DETECTION RESULTS ON DIFFERENT COMBINING OPTIONS. OPT1-OPT10 DENOTE TEN COMBINING OPTIONS, IN WHICH OF1-OF4 ARE FOUR OPTICAL FLOWS, AND AFS DENOTES THE ANCHOR FRAMES SELECTION.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall</td>
<td>0.65</td>
<td>0.73</td>
<td>0.79</td>
<td>0.79</td>
<td>0.81</td>
<td>0.80</td>
<td>0.83</td>
<td>0.84</td>
<td>0.83</td>
<td>0.85</td>
</tr>
<tr>
<td>Pre</td>
<td>0.72</td>
<td>0.74</td>
<td>0.73</td>
<td>0.73</td>
<td>0.75</td>
<td>0.75</td>
<td>0.79</td>
<td>0.78</td>
<td>0.77</td>
<td>0.87</td>
</tr>
<tr>
<td>$F_1$</td>
<td>0.67</td>
<td>0.71</td>
<td>0.76</td>
<td>0.78</td>
<td>0.78</td>
<td>0.81</td>
<td>0.81</td>
<td>0.83</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Opt1 Opt2 Opt3 Opt4 Opt5
OF1 + GMM OF4 + AFS + GMM OF1 + RPCA OF2 + RPCA OF3 + RPCA OF4 + RPCA OF5 + RPCA
Opt6 Opt7 Opt8 Opt9 Opt10
OF4 + RPCA OF1 + AFS + RPCA OF2 + AFS + RPCA OF3 + AFS + RPCA OF4 + AFS + RPCA OF5 + AFS + RPCA

D. Performance of Our Method with Different Combining Options

In this section, we test different combining options to verify the importance of different modules included in our MAMR model. Four optical flow computation methods, i.e., OF1 by Black and Anandan [47], OF2 by Liu et al. [37], OF3 by Sun et al. [48], OF4 by Brox et al. [49], are used to derive the weight matrix. These methods provide different tradeoffs between speeds and accuracy. In total, ten different combining options are designed for comprehensive comparison (shown in Table II). The acronyms of the combining options are also explained in Table II. For Opt1 and Opt2, the compound data of optical flows and pixels values are modeled with GMM, in which the background image is updated on-the-fly, and foreground is detected by comparing its probability belonging to the foreground over that belonging to the background. The quantitative results and visual comparison, are given in Table II and Fig. 7, respectively.

As shown in Table II, different optical flows obtain almost the same results under the same type of combinations (OF + RPCA or OF + AFS + RPCA). Therefore, we choose the fast optical flow algorithm (OF2) [37] to accelerate our method. Comparing the results of OF+RPCA and OF+AFS+RPCA, we observe that the performance would decline if the AFS is excluded, which demonstrates the effectiveness of AFS to detect moving objects.

Observing the effect of the weighting matrix constructed from optical flow with anchor frames selection, one may want to see the effect of using this weighting matrix with other models such as GMM. To this end, we replace the RPCA model with the GMM model in OPT1 and OPT2. As shown in Table II, the replacement of RPCA with GMM suffers from severe performance loss. In Fig. 7, we show the performance evolution in a more intuitive way with visual comparison. Only using GMM on color information cannot estimate the foreground precisely, e.g., the man in Office and the car in Winter. When adding motion information (Opt1), the results are improved, but some regions still cannot be detected due to the failure of frame-by-frame optical flow computation in detecting slowly-moving objects. By further introducing anchor frame selection (Opt2), most pixels of the foreground can be found. However, there are still some smearing artifacts due to the background variations. The results generated by our method (Opt10), shown in Fig. 7(e), are more accurate, and the recovered backgrounds are more close to the groundtruth. Experimental results in this section verify that each module of our method plays an important role in improving the performance, and the assembling of the three components in our method show great power towards accurate background-foreground separation.

E. Experimental Results on Background Extraction

Figure 8 compares backgrounds extracted by SOBS [25], LBG [42], MBS [24], PCP [15], DECOLOR [9], and our MAMR. We test all the video clips, but present the results for only the most challenging seven ones to save space (see the project website for the results on all the video clips). For the same reason, of the five RPCA-based methods, we present the results for only the baseline PCP [15] and the most recent DEOLOR [9]. The results in Fig. 8 show that our method provides significant improvement over other methods. The background images recovered by our MAMR model are more close to groundtruth while the ones extracted by other methods present smearing and ghosting artifacts.

For Boulevard, Fall, Fountain01, and Fountain02, the foreground objects are small and run fast in the scenes. For this type of motions, all the methods can recover promis-
ing background images. However, when it comes to slowly-moving objects, e.g., the walking men in Office, Overpass, and Monitor, and the running boats in Boats and Canoe, results produced by the compared methods present severe smearing artifacts. This is because the slowly-moving objects occlude the scene across many frames, which may be considered as a part of background, resulting in the failure of background extraction. Moreover, for Winter, the left car keeps motionless at first, and moves very slowly during the whole video (nearly camouflage). SOBS, LBG, and MBS tend to classify the intermittent moving object as background and fail to adapt to background changes. The RPCA-based methods, i.e., PCP and DECOLOR, present smearing artifacts along the trajectories of running car. On the contrary, our method achieves promising results for all the evaluation datasets. With the help of motion information, we can prevent the slow moving objects (e.g., motionless man, running boat) from leaking into backgrounds, and recover the accurate backgrounds without smearing and ghosting artifacts.

F. Experimental Results on Foreground Detection

With the extracted background, we detect foreground objects via background subtraction. Foreground detection results are reported in Table III. Our method achieves the highest F-measure for all the datasets, though some values in terms of the Precision and Recall metrics are a little lower than other methods. For FBM, SBL and DECOLOR, the values of Precision and Recall present a trend that if the value of one metric is very high, the other would be very low. For Monitor, SBST achieves the highest Recall (0.97), but extremely low Precision (only 0.42). As a result, these methods have low F-measure values. In contrast, our method obtains high values in terms of both Precision and Recall, and therefore has high F-measure values. This proves the superior performance of our method over other methods.

Figure 9 further presents visual comparison results of fore-
The most difficult category on detecting foreground is the Dynamic Background. As compared to the background in Fall, Overpass, and springs in Fountain01, Fountain02, the judge on whether the pixel belongs to foreground or background is very difficult. For example, in both Shade and Train, all the methods fail to detect the body of the boat while our MAMR model is able to faithfully separate the boat; in Fall, most methods cannot fend against the influence of the waving tree, and the foreground masks are polluted severely. DECOLOR provides comparable results to our methods and ensures the integrity of the foreground, but also yields overestimation in some cases, which can be observed from the man in both Shade and Overpass. Moreover, for Fountain01, the flowing fountain water is misclassified as part of foreground and further expanded due to the smoothness regularization.

In general, our method significantly outperforms other methods. The results of our MAMR model are the closest to ground truth binary maps. Through encoding motion cues into RPCA, our motion-aware method significantly improves the performance of other motion-unaware RPCA methods.

### G. Experimental Results on Noisy Datasets

In this section, we test the performance of our RMAMR model against noisy datasets. To this end, we add Gaussian noise with a variance of 25 to the original test clips. The noise degradation can affect a lot on background extraction and foreground detection.

Objective recovery results of foreground detection and background extraction are reported in Table IV and Table V, respectively. As shown in Table IV, though most methods including ours obtain a lower metric values than results on clean datasets (Table III in Sec. IV-F), our method still obtains the best objective values for most cases, which demonstrates robustness of our RMAMR model to noise. In Table V, our method obtains the highest PSNRs against the groundtruth backgrounds. Note that all the RPCA-based methods achieve satisfactory denoising results, which have relative higher values of PSNR than other methods. Figure 10 further presents visual comparisons of foreground detection results. Our method generates almost the same foreground results as those on clean datasets, while other methods tend to produce noisy results due to the presence of noise.

### TABLE III

Quantitative Foreground Separation Results in Terms of Recall, Precision, and F-Measure on the Twelve Video Clips.

<table>
<thead>
<tr>
<th>Office</th>
<th>Winter</th>
<th>Boulevard</th>
<th>Shade</th>
<th>Monitor</th>
<th>Train</th>
<th>Boats</th>
<th>Canoe</th>
<th>Fall</th>
<th>Fountain01</th>
<th>Fountain02</th>
<th>Overpass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra</td>
<td>Pre</td>
<td>Re</td>
<td>Ra</td>
<td>Pre</td>
<td>F1</td>
<td>Ra</td>
<td>Pre</td>
<td>F1</td>
<td>Ra</td>
<td>Pre</td>
<td>F1</td>
</tr>
<tr>
<td>MAMR</td>
<td>0.85</td>
<td>0.86</td>
<td>0.85</td>
<td>0.68</td>
<td>0.72</td>
<td>0.70</td>
<td>0.68</td>
<td>0.75</td>
<td>0.73</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>Vibe[2]</td>
<td>0.70</td>
<td>0.80</td>
<td>0.69</td>
<td>0.57</td>
<td>0.63</td>
<td>0.52</td>
<td>0.50</td>
<td>0.52</td>
<td>0.54</td>
<td>0.67</td>
<td>0.64</td>
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<tr>
<td>SOBS[25]</td>
<td>0.67</td>
<td>0.79</td>
<td>0.69</td>
<td>0.53</td>
<td>0.43</td>
<td>0.61</td>
<td>0.43</td>
<td>0.78</td>
<td>0.76</td>
<td>0.61</td>
<td>0.97</td>
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<td>GMM[13]</td>
<td>0.53</td>
<td>0.62</td>
<td>0.39</td>
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<td>0.45</td>
<td>0.37</td>
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<td>SBL[36]</td>
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<td>0.96</td>
<td>0.81</td>
<td>0.45</td>
<td>0.57</td>
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<td>0.19</td>
<td>0.73</td>
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<td>DECOLOR[9]</td>
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<td>0.60</td>
<td>0.50</td>
<td>0.50</td>
<td>0.58</td>
<td>0.47</td>
<td>0.44</td>
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<td>0.60</td>
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<tr>
<td>Pre</td>
<td>F1</td>
<td>Re</td>
<td>Ra</td>
<td>Pre</td>
<td>F1</td>
<td>Ra</td>
<td>Pre</td>
<td>F1</td>
<td>Ra</td>
<td>Pre</td>
<td>F1</td>
</tr>
<tr>
<td>RMAMR</td>
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<td>27.12</td>
<td>31.20</td>
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<td>39.87</td>
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<td>MBS[24]</td>
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<td>PC(15)</td>
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<td>21.73</td>
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<td>21.23</td>
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<td>32.50</td>
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<td>SSGoDec[35]</td>
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<td>27.50</td>
<td>35.44</td>
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<td>25.00</td>
<td>28.61</td>
<td>36.32</td>
<td>33.04</td>
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<tr>
<td>DECOLOR[9]</td>
<td>29.83</td>
<td>26.02</td>
<td>31.02</td>
<td>37.20</td>
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</tr>
</tbody>
</table>

### TABLE V

Quantitative Background Extraction Results in Terms of PSNR on Twelve Noisy Video Clips.

<table>
<thead>
<tr>
<th>Office</th>
<th>Winter</th>
<th>Shade</th>
<th>Monitor</th>
<th>Train</th>
<th>Boats</th>
<th>Canoe</th>
<th>Fall</th>
<th>Fountain01</th>
<th>Fountain02</th>
<th>Overpass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra</td>
<td>Pre</td>
<td>Re</td>
<td>Ra</td>
<td>Pre</td>
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<td>Ra</td>
<td>Pre</td>
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<td>33.06</td>
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<td>35.43</td>
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<td>30.93</td>
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<td>21.73</td>
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<td>SSGoDec[35]</td>
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<td>24.19</td>
<td>27.50</td>
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<td>SBL[36]</td>
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<td>25.00</td>
<td>28.61</td>
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<td>DECOLOR[9]</td>
<td>29.83</td>
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Fig. 9. Visual quality comparison for foreground detection on twelve video clips: (a) input image frame and (b) corresponding groundtruth binary foreground, (c) our MAMR model, (d) ViBe [21], (e) SOBS [23], (f) GMM [13], (g) FBM [28], (h) PCP [15], and (i) DECOLOR [9]. From top to bottom present the 656th frame of Office, the 1936th frame of Winter, the 816th frame of Boulevard, the 481th frame of Shade, the 46th frame of Monitor, the 46th frame of Train, the 7101th frame of Boats, the 956th frame of Canoe, the 1497th frame of Fall, the 717th frame of Fountain01, the 741th frame of Fountain02, and the 2401th frame of Overpass, respectively. The gray regions in the ground-truths provided by CDnet are excluded when making objective comparison.

**H. Running Time**

Our method mainly consists of two parts: dense motion estimation by optical flow [37] and convex programming in solving the MAMR/RMAMR models. We report running time for Fountain01 with 40 frames of size 320 × 240. The ADM-ALM algorithms are implemented in MATLAB (R2013a), and run on a desktop with a 3.4 GHz Core4 i7 processor and 8 GB memory.

The motion estimation takes about 20 seconds on average to process 40 frames (each frame takes about 0.5 seconds). The ALM-ADM algorithm takes 2.53 seconds to separate the background and foreground from the 40-frame sequence by solving Model (5), while takes 2.60 seconds solving Model.
TABLE IV

| Office Winter Boulevard Shade Monitor Train Boats Canoe Fall Fountain01 Fountain02 Overpass |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ra Pre F1 Ra Pre F1 Ra Pre F1 Ra Pre F1 Ra Pre F1 Ra Pre F1 Ra Pre F1 Ra Pre F1 Ra Pre F1 |
| RMAMR | 0.66 | 0.64 | 0.64 | 0.79 | 0.67 | 0.70 | 0.78 | 0.70 | 0.79 | 0.70 | 0.61 | 0.81 |
| SOBS[25] | 0.69 | 0.70 | 0.67 | 0.69 | 0.61 | 0.66 | 0.69 | 0.65 | 0.65 | 0.70 | 0.66 | 0.68 |
| LBG[42] | 0.57 | 0.65 | 0.78 | 0.64 | 0.42 | 0.63 | 0.46 | 0.44 | 0.74 | 0.80 | 0.70 | 0.66 |
| MBS[24] | 0.84 | 0.32 | 0.44 | 0.38 | 0.41 | 0.49 | 0.65 | 0.52 | 0.82 | 0.54 | 0.56 | 0.71 |
| PCP[15] | 0.52 | 0.80 | 0.60 | 0.44 | 0.41 | 0.50 | 0.72 | 0.60 | 0.73 | 0.69 | 0.70 | 0.76 |
| OP[34] | 0.32 | 0.58 | 0.43 | 0.30 | 0.20 | 0.37 | 0.48 | 0.40 | 0.58 | 0.47 | 0.53 | 0.69 |
| SSGoDec[35] | 0.62 | 0.74 | 0.63 | 0.60 | 0.51 | 0.63 | 0.60 | 0.53 | 0.56 | 0.71 | 0.72 | 0.73 |
| SBL[36] | 0.57 | 0.72 | 0.60 | 0.65 | 0.63 | 0.60 | 0.70 | 0.79 | 0.70 | 0.70 | 0.62 | 0.68 |
| DECOR[9] | 0.62 | 0.65 | 0.70 | 0.64 | 0.73 | 0.67 | 0.87 | 0.70 | 0.70 | 0.35 | 0.40 | 0.83 |

The proposed framework could be improved and extended in future work: 1) exploit the incorporation of more complex motion models or other clues into the low-rank and sparse recovery framework for foreground detection, 2) optimize model parameters according to video characteristics, 3) explore weighted versions of more low-rank and sparse recovery models as well as their applications to other image processing tasks.

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References


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